Closed Regular Electrode Structure for SAW Resonators

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***Abstract*—A new closed regular electrode structure for future surface acoustic wave (SAW) devices is described. The structure consists of an interdigital transducer in the form of a ring placed on the Z cut of a hexagonal piezoelectric crystal. Finite thickness electrodes produce the known slowing effect for a SAW in comparison with this SAW on a free surface. So, the "slow" electrode region with the "fast" surrounding region forms an open waveguide structure with the acoustic field concentrated in the electrode region. Because of a finite curvature of the waveguide, the propagating SAW mode has unavoidable losses of its energy due to radiation into the surrounding region. However, these losses are decreased exponentially with increasing of waveguide radius, and an acceptable level of radiation losses can be reached. Estimations of the quality factor *Q* in the case of Al electrodes with 5% thickness/wavelength ratio and with five wavelength aperture placed on an AlN substrate show the changing of *Q* from 2×10^2 to 2×10^8 with waveguide radius changing from 50 to 300 wavelengths, respectively. The new structure can be named as ring waveguide resonator (RWR) on SAW. Due to its regularity the electrical admittance of RWR does not have sidelobes, which are typical for usual SAW resonators composed of different electrode gratings with gaps.**

I. Introduction

Recently the authors introduced into consideration a planar ring electrode structure for surface acoustic wave (SAW) applications [1], [2]. Note that two critical structures for the development of surface acoustic wave (SAW) devices have been proposed about 40 years ago [3], [4]. Formed on piezoelectric substrates, an electrode interdigital transducer [3] (IDT) and a reflective grating [4] gave rise to a steady progress in the application of SAW technology to signal processing and to mobile communication systems. In the usual SAW devices the direction of wave propagation is fixed or changed by a discrete angle using the reflective grating. In any SAW device several different gratings with gaps between them need to keep the SAW energy in a restricted region. However, in turn, the irregularities of such structures arouse both parasitic SAW reflections and bulk acoustic wave generation. Presumably, the regular closed electrode structure (Fig. 1) proposed in [1] and [2] is free from such disadvantages. Therefore this structure has to be investigated.

The structure consists of an IDT in the form of a ring (Fig.1) placed on the Z cut of a hexagonal piezoelectric crystal such as aluminum nitride (AlN). On the free surface of such a crystal cut the SAW phase velocity v_f is isotropic. If the SAW velocity under the electrode grating v_s is smaller than v_f then a waveguide effect can be expected. The closed "slow" electrode region with the "fast" surrounding region forms an open waveguide resonator structure excited by the ring IDT. The structure can be termed as a ring waveguide resonator (RWR) on SAW [2]. In the present paper, as a first step towards an adequate model of RWR, a simplified theory is proposed.

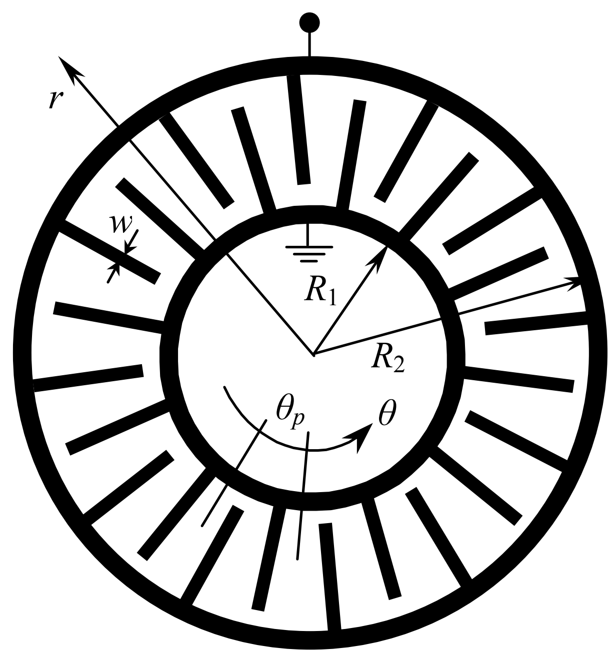

Figure 1. Ring waveguide resonator on SAW.

II. SIMPLIFIED THEORY

Let us introduce in Fig. 1 the following designations: θ and r are the polar coordinates, θ_p is the angular period of IDT, and R_1 and R_2 are the internal and external radii of the electrode ring, respectively. For simplicity, the width of the IDT bus bars is neglected. In order to estimate v_s, suppose also that the ring IDT aperture $A=(R_2-R_1)$ is much smaller then the ring mean radius $R=(R_1+R_2)/2$. In this case the properties of the ring IDT can be estimated from the ones of a usual "in line" IDT with period $p=R\theta_p$ for the given relative electrode thickness h/p and metallization ratio $2w/p$, where, respectively, h and w are the thickness and the width of electrode for $r=R$.

The calculation of a standard set of IDT parameters (the so-called coupling-of-modes parameters) has been done by the known impedance method [5]. For AlN material parameters [6] and Al electrodes with $h/p=0.05$ and $2w/p=0.7$ the following results have been obtained: reflection coefficient per IDT period $\hat{\Gamma}=i\Gamma$, where $\Gamma=-0.032$, transduction coefficient $K^2=0.00036$, capacitance of IDT period per aperture $C=13.4\times\varepsilon_0$, and, finally, velocity $v_s=5579\,m/s$. Note that the velocity $v_f=5770\,m/s$.

Let us now estimate the waveguide effect in the ring structure using an effective approximate method of SAW description based on the scalar two-dimensional wave equation [7], which needs only to be rewritten in polar coordinates r and θ. If $\Psi(r,\theta)$ is the surface normal component of an acoustic displacement field in a propagating waveguide mode, then for the structure with radial symmetry it may be expressed as

$$\Psi(r,\theta)=\psi(r)e^{im\theta-i\omega t}, \tag{1}$$

where m is the angular wave number, ω is the frequency, and t is the time. The amplitude $\psi(r)$ obeys the wave equation

$$r^2\frac{\partial^2\psi}{\partial r^2}+r\frac{\partial\psi}{\partial r}+\left(k^2r^2-m^2\right)\psi=0, \tag{2}$$

where the wave number $k=\omega/v$, the velocity $v=v_s$ for $R_1<r<R_2$, and $v=v_f$ for $0\le r\le R_1$ and $R_2\le r<\infty$. In each range of r the solution of such equation is a cylinder function, which is expressed a linear combination of the Bessel functions of the first kind $J_m(kr)$ and of the second kind $Y_m(kr)$ or of the third kind $H_m^{(1)}(kr)$. For a restricted function, which satisfies the radiation condition at $r=\infty$,

$$\psi(r)=\begin{cases} c_1J_m(k_fr), & 0\le r\le R_1 \\ c_2J_m(k_sr)+c_3Y_m(k_sr), & R_1\le r\le R_2 \\ c_4H_m^{(1)}(k_fr), & R_2\le r<\infty \end{cases}, \tag{3}$$

where $k_f=\omega/v_f$, $k_s=\omega/v_s$, and c_1,c_2,c_3,c_4 are arbitrary constants. The function (3) has to be continuously differentiable at $r=R_1$ and at $r=R_2$. This requirement gives the following linear system of four equations for four constants:

$$\begin{aligned} &c_1J_m(k_fR_1)-c_2J_m(k_sR_1)-c_3Y_m(k_sR_1)=0, \\ &c_1k_fJ'_m(k_fR_1)-c_2k_sJ'_m(k_sR_1)-c_3k_sY'_m(k_sR_1)=0, \\ &c_2J_m(k_sR_2)+c_3Y_m(k_sR_2)-c_4H_m^{(1)}(k_fR_2)=0, \\ &c_2k_sJ'_m(k_sR_2)+c_3k_sY'_m(k_sR_2)-c_4k_fH_m'^{(1)}(k_fR_2)=0, \end{aligned} \tag{4}$$

where the primed symbols indicate corresponding derivatives of Bessel functions with respect to their arguments. A dispersion equation of waveguide modes is then as

$$\Delta(m,\omega)=0, \tag{5}$$

where $\Delta(m,\omega)$ is the determinant of system (4). From Eqs. (3)-(5) both the angular wave number m and the amplitude $\psi(r)$ can be determined. Due to condition (5) the system (4) contains an arbitrary constant. For the sake of definiteness in calculation of function $\psi(r)$ below it is set that $c_1=1$. Note here that the propagation of a pure waveguide mode (that is without attenuation) is impossible in principle. It follows from a simple reasoning. If in Eq. (1) the angular wave number m is real, then the corresponding wave number on the distance r is equal to $k_r=m/r$ and it decreases with radius. So, it becomes smaller than the SAW wave number on a free surface $k_f=\omega/v_f$ for radius $r>m/k_f$, and the known Cherenkov radiation of energy arises. As a result, the angular wave number should be complex, $m=m'+im''$, with imaginary part $m''>0$ to agree with a loss of the mode energy into the external region $r>R_2$ with θ increasing.

III. NUMERICAL RESULTS

In Fig. 2 and Fig. 3 the dependences of real and of imaginary parts of the angular wave number m as a function of R/λ are shown, respectively, where

$$\lambda=2\pi v_f/\omega \tag{6}$$

is the wavelength of SAW on a free surface. The normalized waveguide aperture $A/\lambda=5$ was used for calculations. The corresponding quality factor of RWR as

$$Q=m'/(2m'') \tag{7}$$

is shown in Fig. 3 also. It changes from 2×10^2 to 2×10^8 with waveguide radius changing from 50 to 300 wavelengths, respectively.

In Figs. 4-6 the field complex amplitude $\psi(r)=\psi'(r)+i\psi''(r)$ in the cases of the mean ring radius $R/\lambda=$ 50, 100, and 200 as a function of radial coordinate r/λ is shown, respectively, if the aperture $A/\lambda=5$. It can be seen, that the field is concentrated in the slowing region $R_1<r<R_2$ with its magnitude maximum shifted from the ring mean radius R towards the external radius

$R_2 = R + A/2$. It should be mentioned here that a nonzero field outside the RWR can be used for a possible coupling with other electrode structures as it might be of interest for future applications. The magnitude $|\psi(r)|$ of this field (if $r > R_2$), with respect to the magnitude maximum into the ring, fast decreases with the ring radius R/λ increasing and it can be controlled by this parameter (Figs. 4-6).

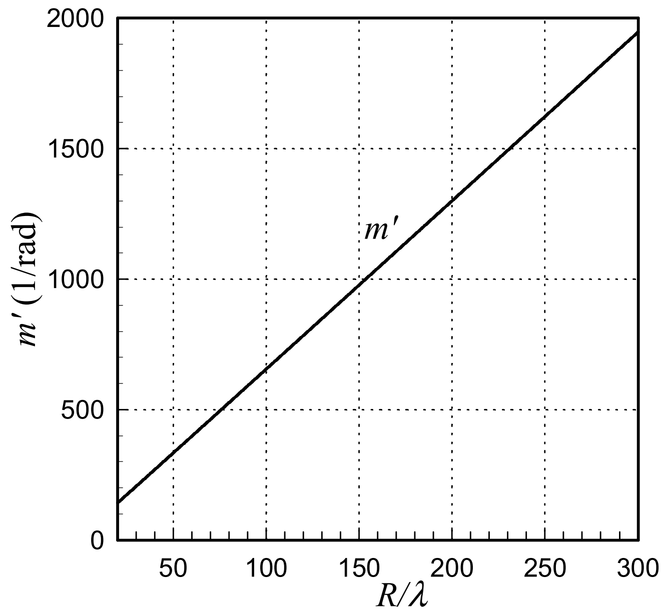

Figure 2. Real part of the angular wave number, A/λ=5.

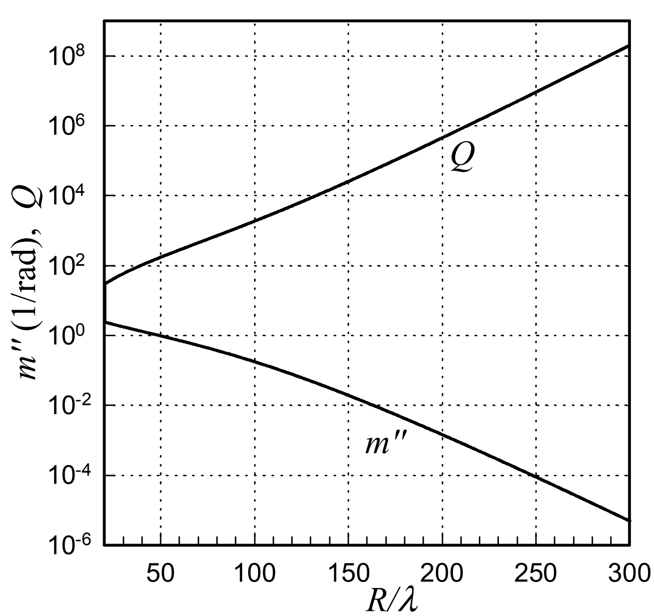

Figure 3. Imaginary part of the angular wave number and the quality factor of RWR, A/λ=5.

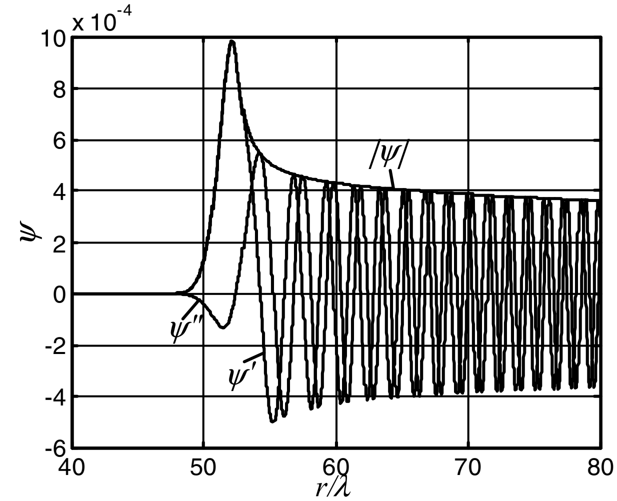

Figure 4. Acoustic field distribution in RWR, R/λ=50 and A/λ=5.

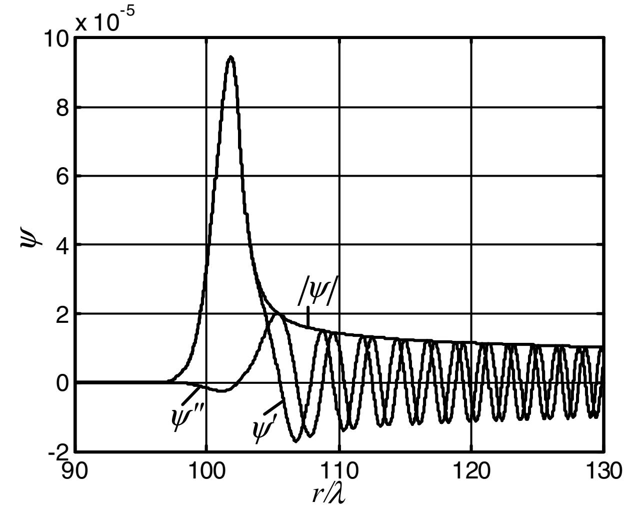

Figure 5. Acoustic field distribution in RWR, R/λ=100 and A/λ=5.

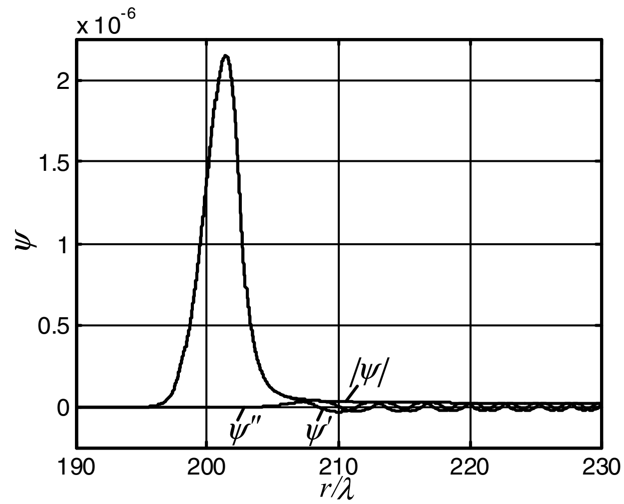

Figure 6. Acoustic field distribution in RWR, R/λ=200 and A/λ=5.

IV. Electrical Admittance

Let us estimate now the electrical properties of the RWR (Fig. 1). Because its structure is not changed along the polar coordinate θ, the wave fields in each period are repeated. Such properties in the case of a usual IDT correspond to the IDT of the infinite length, for which there is a simple expression [8] for the electrical input admittance. By analogy, it is possible to give a corresponding expression for the ring case replacing there, if necessary, parameters for the Cartesian coordinates with parameters for the polar coordinates. So, the following expression for the input electrical admittance of RWR per period and per aperture can be written:

$$Y = -i\omega C\left(1 - \frac{8K^2}{\pi}\frac{1}{m\theta_p - 2\pi + \Gamma}\right), \qquad (8)$$

where $m = m' + im''$ and all parameters are explained above. Because of the reflection coefficient $|\Gamma| << 2\pi$, the resonance properties of Y become apparent near the Bragg condition $m'\theta_p = 2\pi$. In order to see the frequency dependence explicitly, the real part of the angular wave number (Fig. 2) can be represented as

$$m' = \omega/\Omega , \qquad (9)$$

where by the dispersion of a mode angular velocity Ω is neglected. Near the resonance the radiation losses are described in Eq. (8) by the value of $m''\theta_p = \pi/Q$. The frequency dependence of the complex admittance $Y = Y' + iY''$ for $R/\lambda = 200$ is shown in Fig. 7, exhibiting a perfect form. Due to Eq. (6) the frequency ω by ratio

$$\frac{R\theta_p\omega}{2\pi v_f} = \frac{R\theta_p}{\lambda} \qquad (10)$$

is related with a dimensionless normalized frequency, which is used on abscissa in Fig. 7.

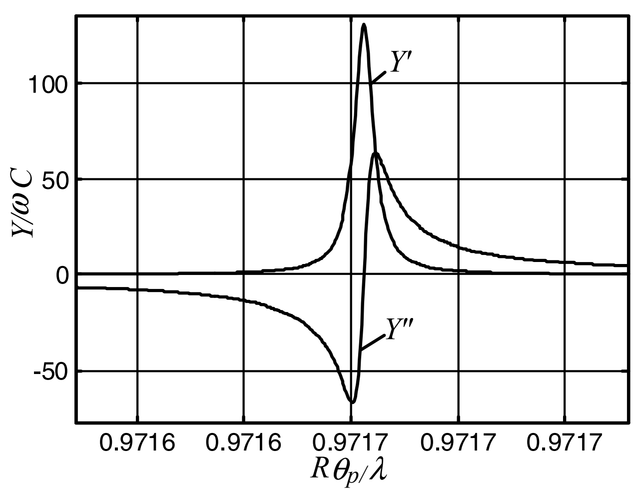

Figure 7. Input electrical admittance of RWR, R/λ=200 and A/λ=5.

V. Conclusion

Properties of a ring waveguide resonator on SAW with high quality factor are estimated. Due to its regularity the electrical admittance of RWR does not have sidelobes, which are typical for the usual SAW resonators composed of different finite electrode gratings with gaps.

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